Recall that “real” amplifiers are only approximately linear!

If the input signal becomes too large, and/or the input signal changes too quickly, we begin to see some very non-linear behavior.

Non-linear behavior leads to a distorted output.

In other words, the output does not look like a copy of the input!

The input signal cannot be too big:

**HO: OUTPUT VOLTAGE SATURATION**

The input signal cannot change too fast:

**HO: SLEW RATE**

The input signal certainly cannot be too be and change too fast!

**HO: FULL POWER BANDWIDTH**
Recall that the **ideal** transfer function implies that the **output voltage** of an amplifier can be **very** large, provided that the gain $A_{vo}$ and the input voltage $v_{in}$ are large.

![Graph showing output voltage saturation]

- $A_{vo} < 0$
- $A_{vo} > 0$
The output voltage is limited

However, we found that in a “real” amplifier, there are limits on how large the output voltage can become.

The transfer function of an amplifier is more accurately expressed as:

\[
\begin{align*}
v_{\text{out}}(t) = \begin{cases} 
L_+ & v_{\text{in}}(t) > L_+^{\text{in}} \\
A_v v_{\text{in}}(t) & L_-^{\text{in}} < v_{\text{in}}(t) < L_+^{\text{in}} \\
L_- & v_{\text{in}}(t) < L_-^{\text{in}}
\end{cases}
\end{align*}
\]
A non-linear behavior!

This expression is shown graphically as:

This expression (and graph) shows that electronic amplifiers have a maximum and minimum output voltage ($L_+$ and $L_-$).

If the input voltage is either too large or too small (too negative), then the amplifier output voltage will be equal to either $L_+$ or $L_-$. If $v_{out} = L_+$ or $v_{out} = L_-$, we say the amplifier is in saturation (or compression).
Make sure the input isn’t too large!

Amplifier saturation occurs when the input voltage is greater than:

\[ v_{in} > \frac{L_+}{A_v} = L_{in}^+ \]

or when the input voltage is less than:

\[ v_{in} < \frac{L_-}{A_v} = L_{in}^- \]

Often, we find that these voltage limits are symmetric, i.e.:

\[ L_+ = -L_- \quad \text{and} \quad L_{in}^+ = -L_{in}^- \]

For example, the output limits of an amplifier might be \( L_+ = 15 \text{ V} \) and \( L_- = -15 \text{ V} \).

However, we find that these limits are also often asymmetric (e.g., \( L_+ = +15 \text{ V} \) and \( L_- = +5 \text{ V} \)).
Saturation: Who really cares?

Q: Why do we care if an amplifier saturates? Does it cause any problems, or otherwise result in performance degradation?

A: Absolutely! If an amplifier saturates—even momentarily—the unavoidable result will be a distorted output signal.
A distortion free example

For example, consider a case where the input to an amplifier is a triangle wave:

Since $L_{in}^+ < v_{in}(t) < L_{in}^-$ for all time $t$, the output signal will be within the limits $L_+$ and $L_-$ for all time $t$, and thus the amplifier output will be $v_{out}(t) = A_{vo} v_{in}(t)$:
The input is too darn big!

Consider now the case where the input signal is much larger, such that 
\( v_{in}(t) > L^+_{in} \) and \( v_{in}(t) < L^-_{in} \) for some time \( t \) (e.g., the input triangle wave exceeds the voltage limits \( L^+_{in} \) and \( L^-_{in} \) some of the time):

\[
\begin{align*}
\text{This is precisely the situation about which I earlier expressed caution.} \\
\text{We now must experience the palpable agony of signal distortion!}
\end{align*}
\]
Note that this output signal is **not** a triangle wave!

For time $t$ where $v_{in}(t) > L_+^{in}$ and $v_{in}(t) < L_-^{in}$, the value $A_v v_{in}(t)$ is greater than $L_+$ and less than $L_-$, respectively.

Thus, the output voltage is limited to $v_{out}(t) = L_+$ and $v_{out}(t) = L_-$ for these times.

As a result, we find that output $v_{out}(t)$ does **not** equal $A_v v_{in}(t)$ — the output signal is **distorted**!
Amplifiers with op-amps

For amplifiers constructed with op-amps, the voltage limits \( L_+ \) and \( L_- \) are determined by the DC Sources \( V^+ \) and \( V^- \):

\[
L_+ \approx V^+ \quad \text{and} \quad L_- \approx V^-
\]
Slew Rate

We know that the output voltage of an amplifier circuit is limited, i.e.:

\[ L^- < v_{out}(t) < L^+ \]

During any period of time when the output tries to exceed these limits, the output will saturate, and the signal will be distorted! E.G.:
Limits on the time derivative

But, this is **not** the only way in which the output signal is limited, **nor** is saturation the only way it can be **distorted**!

A very important op-amp parameter is the **slew rate** (S.R.).

Whereas $L_-$ and $L_+$ set limits on the values of output signal $v_{out}(t)$, the slew rate sets a limit on its **time derivative** !!!! I.E.:

$$-S.R. < \frac{d v_{out}(t)}{dt} < +S.R.$$

In other words, the output signal can **only change so fast**! Any attempt to exceed this fundamental op-amp limit will result in **slew-rate limiting**.
The red means distortion

So, in addition to saturation:

\[
v_{out}(t) = \begin{cases} 
L & \text{if} \quad A_v \cdot v_{in}(t) > L_+ \\
A_v \cdot v_{in}(t) & \text{if} \quad L_- < A_v \cdot v_{in}(t) < L_+ \\
L & \text{if} \quad L > A_v \cdot v_{in}(t)
\end{cases}
\]

we find the following output signal condition:

\[
v_{out}(t) = \begin{cases} 
A_v \cdot v_{in}(t) & \text{if} \quad \left| \frac{d}{dt} A_v \cdot v_{in}(t) \right| < S.R. \\
\pm (S.R.) t + C & \text{if} \quad \left| \frac{d}{dt} A_v \cdot v_{in}(t) \right| > S.R.
\end{cases}
\]
For example

For example, say we build a non-inverting amplifier with mid-band gain $A_v = 2$.

This amplifier was constructed using an op-amp with a slew rate equal to $4V/\mu\text{sec}$.

**Q:** If we input the following signal $v^\text{in}(t)$, what will we see at the output of this amplifier?

![Graph of the input signal $v^\text{in}(t)$ with a step from -2 to 2 volts over a time period of 12 microseconds, with a rise time of 4 microseconds.](image)
This is what it should look like

**A:** Ideally, the output would look exactly like the input, only multiplied by $A_v = 2$:

$$v_{out}(t) = 2v_{in}(t) \quad \text{(ideal)}$$

Note that the time derivative of this output is **zero** at **almost** every time $t$:

$$\frac{dv_{out}(t)}{dt} = 0 \quad \text{for almost all time} \ t$$
Now you see the problem!

The exceptions are at times $t=4$, $t=8$, and $t=12 \ \mu\text{sec}$, where we find that the time derivative is infinite!

$$\frac{dv_{out}(t)}{dt} = \infty \quad \text{at times } t = 4 \text{ and } t = 12$$

and

$$\frac{dv_{out}(t)}{dt} = -\infty \quad \text{at time } t = 8$$

This is a problem!

$$\left|\frac{dv_{out}(t)}{dt}\right| = \infty > 4V/\mu\text{sec} !!!$$
This is what it actually looks like!

Thus, the output signal **exceeds** the slew rate of the op-amp—or at least, it **tries** too!

The reality is that since the op-amp output **cannot** change at a rate greater than \( \pm 4V/\mu\text{sec} \), the output signal will be **distorted**!

Note the derivative of the **actual** output signal is limited to a maximum value \( (\pm 4V/\mu\text{sec}) \) by the op-amp **slew rate**.
Consider now the case where the input to an op-amp circuit is sinusoidal, with frequency $\omega$.

The output will thus likewise be sinusoidal, e.g.:

$$v_{out}(t) = V_o \sin \omega t$$

where $V_o$ is the magnitude of the output sine wave.

**Q:** Under what conditions is this output signal possible? In other words, might this output signal be distorted?

**A:** First, the output will **not** be saturated if:

$$V_o \leq L_+ \approx V^+ \quad \text{and} \quad -V_o \geq L_- \approx V^-.$$
The time derivative

Q: So, the output will not be distorted if the above statement is true?

A: Be careful!

It is true that the output will not saturate if magnitude of the sinewave is smaller than the saturation limits.

However, this is not the only way that the signal can be distorted!

Q: I almost forgot! A signal can also be distorted by slew-rate limiting. Could this problem possibly affect a sine wave output?

A: Recall that the slew rate is a limit on the time derivative of the output signal.

The time derivative of our sine wave output is:

\[
\frac{dv_{\text{out}}(t)}{dt} = \omega V_o \cos \omega t
\]
The max and min

Note that the time derivative is proportional to the signal frequency $\omega$.

Makes sense!

As the output signal frequency increases, the output voltage changes more rapidly with time.

Also note however, that this derivative is a likewise a function of time. The maximum value occurs when $\cos \omega t = 1$, i.e.,

$$\left. \frac{dv_{out}(t)}{dt} \right|_{\text{max}} = \omega V_o$$

while the minimum value occurs when $\cos \omega t = -1$, i.e.,

$$\left. \frac{dv_{out}(t)}{dt} \right|_{\text{min}} = -\omega V_o$$

Thus, we find that the output signal will not be distorted if these values are within the slew rate limits of the op-amp.
A simple way to determine you are slew rate limited

In other words, to avoid distortion by slew rate limiting, we find:

\[ \omega V_o \leq S.R. \]

and

\[ -\omega V_o \geq -S.R. \]

Note that:

1) These two equations are equivalent!

2) The conditions that cause slew-rate distortion depend on both the magnitude \( V_o \) and the frequency \( \omega \) of the output signal!
The frequency can only be so large

Now, recall that there are limits on the magnitude alone, that is:

\[ V_o \leq L \approx V^+ \]

to avoid saturation.

Let’s assume that the output sine wave is as large as it can be without saturating, i.e., \( V_o = V^+ \) and thus:

\[ v_{out}(t) = V^+ \sin \omega t \]

We then find to avoid slew-rate limiting:

\[ \omega V^+ < S.R. \]

Rearranging, a limit on the maximum frequency for this sine wave output (one with maximum amplitude) is:

\[ \omega < \frac{S.R.}{V^+} \cdot \omega_M \]
**Full-Power bandwidth**

The value:

\[ \omega_M = \frac{SR}{V^+} \]

is called the full-power bandwidth of the op-amp (given a DC supply \( V^+ \)).

It equals the largest frequency a full-power (i.e., \( V_o = V^+ \)) sine wave can obtain without being distorted by slew rate limiting!

Thus, if the input signal to \( \omega V^+ < S.R. \) an op-amp circuit is a sine wave, we **might** have to worry about slew rate limiting, if the signal frequency is greater than the full-power bandwidth (i.e., \( \omega > \omega_M \)).
I’ll find out from the exam if you read this page

Please note these **important facts** about full-power bandwidth:

1) The analysis above was performed for a *sine wave* signal. It is explicitly accurate **only** for a sine wave signal. For some other signal, you must determine the time derivative, and then determine its maximum (or minimum) value!

2) Full-power bandwidth is **completely different** than the closed-loop amplifier bandwidth. For example, a signal with a frequency greater than the closed-loop amplifier bandwidth will **not** result in a distorted signal!

3) Distortion due to slew-rate limiting depends **both** on signal amplitude $V^+$ and signal frequency $\omega$. Thus, as sine wave whose frequency is much greater than the full-power bandwidth (i.e., $\omega \gg \omega_M$) may be **undistorted** if its amplitude $V^+$ is sufficiently small.